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ON TWO-DIMENSIONAL ELECTRO-GASDYNAMIC FLOWS WITH ALLOWANCE FOR THE INERTIA OF CHARGED PARTICLES

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Electro-gasdynamic flows in inertia-free approximation and those with allowance for inertia forces are investigated. Conditions under which inertia effects are considerable, are determined. Simple analytical solutions are derived for systems of electro-gasdynamic equations that describe the motion of particles in a uniform external electric field in the presence of tangential discontinuity of gasdynamic velocity at the half-plane boundary. The possibility of reverse current generation, i.e. of the return of particles to the emitter is demonstrated. Obtained results are compared with data related to inertia-free approximation. A numerical method is developed for solving the complete system of equations of electro-gasdynamics with allowance for particle inertia. The proposed method is used for investigating the expansion of electro-gasdynamic streams in channels. Results of numerical calculations for various values of controlling parameters are presented. Effects of inertia are set appart.

In many applications (such as electron-ion technology, electrically charged jet streams of aircraft engines) solid or fluid particles in a gasdynamic stream become electrically charged, and it is necessary to investigate two-phase electro-gasdynamic flows. General equations that define the electro-gasdynamic flow of a mixture of inert gas, particles, and ions appear in [1].

If the charged particle inertia is small, two-phase flows can be investigated by the method developed for solving equations of electro-gasdynamics with the Ohm law formulated in the inertia-free approximation [2 — 4]. Investigation of such two-dimensional flows has elucidated a number of interesting qualitative relationships pertaining to the widening of electric streams, reverse currents, etc. [3, 4]. The analysis of spatial (in the simplest case, two-dimensional) flows is much more complex when the inertia of charged particles is taken into account; publications in that field are virtually nonexistent. Below we present some results of an investigation of two-dimensional electro-gasdynamic flows with allowance for charged particle inertia.

1. General equations for the flow of a mixture of inert gas, charged particles, and ions (phases *1*—*3*) are complex [1]. For specific applications the system can be considerably simplified. It is assumed below that phase transition (gas *1* and liquid particles *2*) is absent, the heat flux vector is zero (which does not inhibit heat exchange between phases), the difference between chemical potentials of phases is negligible, and that all particles are of the same size and carry the same charge.

The momentum and continuity equations for phases *2* and *3*, and the Maxwell equations are of the form

$$d\mathbf{V}_2 / dt = k(\mathbf{V}_1 - \mathbf{V}_2) / \rho_2 + q_2\mathbf{E} / \rho_2 + \mathbf{A}, \quad \mathbf{A} = -\nabla p / \rho_2^\circ \qquad (1.1)$$

$$\mathbf{j}_3 = q_3\mathbf{V}_3 = q_3(\mathbf{V}_1 + b_3\mathbf{E})$$

$$\partial\rho_2 / \partial t + \operatorname{div}(\rho_2\mathbf{V}_2) = 0, \qquad \rho_2 = m_2 n_2$$

$$\partial q_3 / \partial t + \operatorname{div}(q_3\mathbf{V}_3) = 0$$

$$\operatorname{rot}\mathbf{E} = 0, \quad \operatorname{div}\mathbf{E} = \frac{4\pi}{\varepsilon}(q_3 + Zen_2)$$

$$k = {}^1/_2\, \rho_1^\circ \pi a^2 n_2 v_{12} c_f(R), \quad R = 2\rho_1^\circ v_{12} a / \mu_1, \quad v_{12} = |\mathbf{V}_1 - \mathbf{V}_2| \qquad (1.2)$$

$$(k = 6\pi a \mu_1 n_2 \quad \text{for} \quad R \ll 1)$$

where $\mathbf{V}_i$ and ρ_i are the phase velocity and density ($i = 1, 2, 3$); m_i and n_i are, respectively, the mass of a single particle and the particle concentration in phase i, q_3 is the volume electric charge of phase *3*, $q_2 = Zen_2$ is the volume electric charge of particles (e is the electron charge, Z is the number of unit charges on a particle); ρ_2° is the density of the particle material; b_3 is the ion mobility; p is the gasdynamic pressure; $\mathbf{E}$ is the electric field, and ε is the medium permittivity. Formulas (1.2) define the law of resistance of particle motion relative to the gas *1*. In these formulas ρ_1° is the true density of gas, μ_1 is the coefficient of gasdynamic viscosity, $c_f = c_f(R)$ is the drag coefficient, and R is the Reynolds number.

In addition to (1.1) and (1.2) the complete system of equations contains the equation of momenta for the inert gas *1*, equations of energy for all three phases, and the thermodynamic relationships between parameters. The first and second of Eqs. (1.1) are formulated on the assumption that only friction forces generated by the inert gas *1* act on particles and ions.

We define such flows in terms of the following dimensionless parameters:

$$\delta = \rho_2 / \rho_2^\circ = m_2 n_2 / \rho_2^\circ, \quad N_2 = \rho_2 / \rho_1 \qquad (1.3)$$

$$N_3 = q_3 EL / (\rho_1 v_1^2), \quad N_4 = q_2 E / (k v_*)$$

$$\tau = \tau_p / T \, (\tau_p = \rho_2 / k, \quad T = L / v_*),$$

where L and v_* are the characteristic length and the relative velocity of phases *1* and *2*, respectively, and δ defines the relative volume occupied by particles (the volume occupied by ions is assumed negligible). If $\delta \ll 1$, the volume of particles can be neglected (then $\rho_1 = \rho_1^\circ$), and the quantity $\mathbf{A}$ in the first of Eqs. (1. 1) is small in comparison with other terms.

Parameter N_2 defines the ratio of the particle friction force acting on gas *1* to the inertia forces of that gas (under condition that $|(\mathbf{V}_1 \nabla) \mathbf{V}_1| \sim |(\mathbf{V}_2 \nabla) \mathbf{V}_2|$). If parameter N_2 is small, the effect of particles on the motion of the inert gas can be neglected. And, if in addition the parameter of electro-gasdynamic interaction N_3, which is the ratio of the friction force exercised by ions *3* on gas *1* to the inertia terms, is also small, the gas flow is determined by conventional equations of gasdynamics. In that case the determination of parameters of phases *2* and *3* can be made on the assumption that the gasdynamic velocity field is known.

Parameter τ is the ratio of the relaxation time τ_p (the time during which velocities of particles *2* and gas *1* are equalized) to the characteristic time T of the problem. If $\tau \ll 1$, the inertia of particles can be neglected, while for $\tau \gg 1$ the friction between particles and gas does not affect the velocity of particles (frozen flow).

Finally, the quantity N_4 defines the ratio of electrostatic forces to friction forces in the particle equation of motion (1. 1).

Let us consider the flow of a mixture for the following parameters:

$$\begin{gathered} a = 10^{-3}\text{ cm}, \quad L = 10\text{ cm}, \quad \mu_1 = 1.78\cdot 10^{-4}\text{ g/(cm. sec)} \\ v_* = 10^4 \text{ cm/sec}, \quad v_1 \approx v_*, \quad n_2 \sim 10^4\text{cm}^{-3}, \quad n_3 \approx 10^9\text{cm}^{-3} \\ \rho_1^\circ = 1.225\cdot 10^{-3}\text{ g/cm}^3, \quad \rho_2^\circ = 1\text{ g/cm}^3, \quad E \sim 10^4\text{ V/cm} \end{gathered} \tag{1.4}$$

These values of parameters relate to the flow of a mixture of inert gas, liquid particles of $10\,\mu$ radius, and ions through an electric field, for instance, in the region of a corona discharge. These values are obtained experimentally in electro-gasdynamic installations.

To determine parameters (1. 3) it is necessary to estimate the electric charge concentrated on a liquid drop. Assuming that drops are charged by the precipitation of ions on their surface owing to the generation of a polarization field, for the maximum charge on a drop we obtain the estimate (Potenier's formula [5])

$$eZ_{\max} = 3\varepsilon E a^2 \tag{1.5}$$

Assuming that ions carry a charge e, we obtain

$$q_3 = en_3 \tag{1.6}$$

Using (1. 3) — (1. 6) we find

$$\begin{gathered} Z = 2.62\cdot 10^6, \quad q_2 = 1 \text{ un. } CGSE\text{ /cm}^3 \\ q_3 = 0.48 \text{ un. } CGSE\text{ /cm}^3, \quad \delta = 4.18\ 10^{-5}, \quad \tau = 1.25 \\ N_2 = 3.42\cdot 10^{-2}, \quad N_3 = 1.31\ 10^{-3}, \quad N_4 = 0.1 \end{gathered} \tag{1.7}$$

where τ is determined in the Stokes approximation.

The estimates of parameters show that: $\rho_1 \approx \rho_1^\circ$, the quantity $\mathbf{A}$ in the first of Eqs. (1. 1) can be neglected, the inertia of the charged particles is appreciable, and that the distribution of gasdynamic parameters is determined by conventional equations of gasdynamics. The latter makes it possible to consider $\mathbf{V}_1$, $\rho_1 = \rho_1^\circ$, μ_1 and b_3 as known. The system of Eqs. (1. 1) and (1. 2) is closed and its solution permits the determination of $\mathbf{V}_2$, q_2, q_3 and $\mathbf{E}$.

Stationary electro-gasdynamic equations with conditions (1. 7) are of the form

$$(\mathbf{V}_2\nabla)\mathbf{V}_2 = K(\mathbf{V}_1 - \mathbf{V}_2) + \varkappa\mathbf{E};\ \varkappa = Ze / m_2;\quad a,\ m_2,\ Z = \text{const} \quad (1.8)$$
$$K = {}^1/_2\rho_1{}^\circ\pi a^2 v_{12} c_f(R) / m_2 \quad (K = 6\pi a\mu_1 / m_2 \quad \text{for} \quad R \ll 1)$$
$$\operatorname{div} q_2\mathbf{V}_2 = 0,\quad \operatorname{div} q_3(\mathbf{V}_1 + b_3\mathbf{E}) = 0$$
$$\operatorname{div}\mathbf{E} = 4\pi(q_2 + q_3) / \varepsilon,\quad \mathbf{E} = -\nabla\varphi$$

where φ is the electric potential.

When the stream is free of ions, the system of equations defining the motion of an inert gas and charged particles becomes

$$(\mathbf{V}_2\nabla)\mathbf{V}_2 = K(\mathbf{V}_1 - \mathbf{V}_2) + \varkappa\mathbf{E},\quad \operatorname{div} q_2\mathbf{V}_2 = 0,\quad \Delta\varphi = -\frac{4\pi}{\varepsilon} q_2 \quad (1.9)$$

This system consists of equations of the elliptic kind with respect to the electric potential and of the hyperbolic kind with respect to velocity $\mathbf{V}_2$ and charge q_2. The trajectories of charged particles are the characteristics.

The analysis of systems (1.8) and (1.9) necessitates the following boundary conditions:

$$\mathbf{V}_2 = \mathbf{V}_{20},\quad q_2 = q_{20},\quad q_3 = q_{30},\quad \varphi = \varphi_0 \ \text{on}\ \Gamma^\circ,\quad \varphi = \varphi_+ \ \text{on}\ \Gamma^{\circ\circ} \quad (1.10)$$

where Γ° is the surface (or line) on which the distribution of parameters at entry to the investigated flow zone is fixed. Only the electric potential is specified on surface $\Gamma^{\circ\circ}$; the distribution of $\mathbf{V}_2$, q_2 and q_3 on $\Gamma^{\circ\circ}$ is provided by the solution of the problem (the gas stream is directed toward $\Gamma^{\circ\circ}$). Instead of specifying the potential on surfaces Γ° and $\Gamma^{\circ\circ}$ it is possible to formulate conditions defined by a combination of the potential φ and its derivatives. Only one such relationship needs to be investigated on each surface. The condition at infinity (only when Γ° and $\Gamma^{\circ\circ}$ taken as a whole do not form a closed surface) must be added to the system of Eqs. (1.10).

Certain solutions of system (1.9),(1.10) for the drag of particles are presented below in the Stokes approximation.

2. Formation of reverse currents. Let the distribution of gasdynamic velocity be of the form (see Fig. 1)

$$\mathbf{V}_1 = (U = \text{const},\ 0,\ 0),\quad y \leqslant 0;\quad \mathbf{V}_1 \equiv 0,\quad y > 0 \quad (2.1)$$

Positively charged particles at velocity

$$\mathbf{V}_2 = \mathbf{V}_{20} = (Uu_0,\ Uv_0,\ 0) \quad (2.2)$$

are introduced into the stream along the line $x = 0,\ y \leqslant 0$.

The stream flows in the external electric field

$$\mathbf{E} = (E_* E_x,\ E_* E_y,\ 0),\quad E_* = \text{const} \quad (2.3)$$
$$E_x = \text{const} < 0,\quad E_y = \text{const} > 0$$

We assume that the electric self-fields are due to the introduction of charged particles into the stream and are considerably smaller than the applied fields. In such case the velocity distribution $\mathbf{V}_2 = (Uu,\ Uv,\ 0)$ in region $x > 0$ is defined by the following equations:

$$\tau du / dt = u_\infty - u,\quad \tau dv / dt = v_\infty - v \quad (2.4)$$
$$\tau = U / (LK),\quad N_4 = E_* Ze / (m_2 UK)$$

$$u_\infty = \begin{cases} 1 + u_\infty', & y < 0 \\ u_\infty', & y > 0 \end{cases} \quad (u_\infty' = N_4 E_x,\ v_\infty = N_4 E_y)$$

where t and coordinates x and y are related to quantities L / U and L, respectively, where L is a characteristic dimension of the flow region.

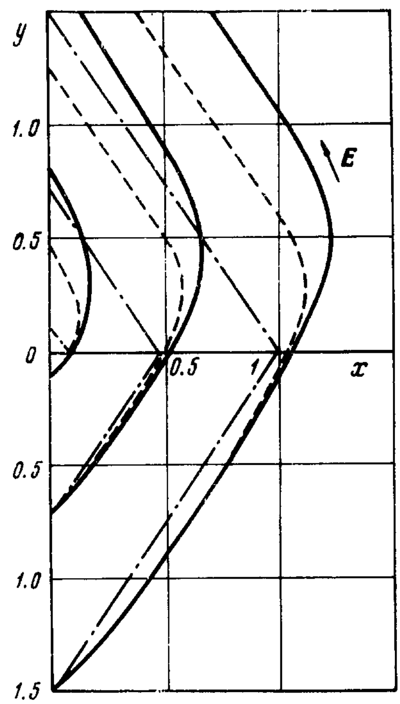


Fig. 1

Quantity K corresponds to the Stokes law of drag. The problem is defined by five parameters: τ, u_0, v_0, u_∞' and v_∞. System (2.4) is integrated in quadratures.

The trajectories of charged particles are shown in Figs. 1 and 2 for $\tau = 1$, 0.5, and 0 by solid, dash and dash-dot lines, respectively, with $u_0 = v_0 = 0.25$. The presence of the retarding electric field causes a bending of trajectories in the region $y > 0$, where gasdynamic velocity is absent, and their return to the plane $x = 0$. When the transverse electric field is zero (Fig. 2) charged particles can reach the plane $y = 0$ only in the presence of initial transverse velocity ($v_0 > 0$). We then have trajectories *1* and *2* (for $\tau = 0{,}5$ and 1, respectively) which separate particles that never approach the boundary $y = 0$ from those that reach the boundary and then return to the plane $x = 0$ owing to the effect of field $E_x < 0$.

Parameter τ defines the inertia of charged particles. For $\tau = 0$ the particle trajectories are straight lines and the initial conditions are immaterial.

3. The flow in a plane channel with conducting walls. Let us consider the stationary motion of charged particles in a semi-infinite space between two parallel grounded walls $y = \pm L / 2$, $x > 0$. The velocity of gas flowing in that region is $\mathbf{V}_1 = (U = \text{const},\ 0,\ 0)$.

Charged particles are introduced into the gas stream along the line $x = 0, |y| \leqslant h^\circ/2$, where $h^\circ < L$, which is a grid electrode at potential $\varphi = 0$. Constant density and initial velocity of charged particles $q > 0$ and $\mathbf{V}_2 = (Uu_0 = \text{const},\ 0,\ 0)$ are specified in this section of the electrode. Since the problem is symmetric with respect to the axis $y = 0$, only the channel upper half $y \geqslant 0$ is considered.

Unlike in the problem investigated in Sect. 2, the particles are subjected to (induced) electric self-fields.

Thus the system of Eqs. (1.9) and (1.10) that determines particle velocity, the electric volume charge, and also the electric fields is of the form

$$\tau\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = 1 - u - \frac{\partial \varphi}{\partial x}, \quad \tau\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -v - \frac{\partial \varphi}{\partial y} \quad (3.1)$$

$$u\frac{\partial q}{\partial x}+v\frac{\partial q}{\partial y}=-q\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right),\quad \Delta\varphi=-q \tag{3.2}$$
$$\tau=m_2U\,/\,(6\pi aL\mu_1)$$

Equations (3.1) and (3.2) are presented in dimensionless form with the following characteristic quantities (denoted by an asterisk):

$$x_*=y_*=L,\quad v_*=U,\quad \varphi_*=UL\,/\,b_*,\quad E_*=U\,/\,b_* \tag{3.3}$$
$$q_*=\varepsilon U\,/\,(4\pi Lb_*),\quad b_*=\varkappa m_2\,/\,(6\pi a\mu_1)$$

The boundary conditions for this system are

$$y={}^1/_2,\quad x\geqslant 0:\ \varphi=0;\quad x=0:\ \varphi=0;\ x\to\infty:\ \varphi\to 0 \tag{3.4}$$
$$y\leqslant h\,/\,2,\quad x=0:\ q=\beta,\ u=u_0,\quad v=0$$
$$y=0,\quad x\geqslant 0:\ \partial\varphi\,/\,\partial y=0$$

The problem is defined by four parameters: τ, u_0, $h=h^\circ\,/\,L$ and $\beta=q_0 4\pi b_* L\,/\,(\varepsilon U)$.

The formulated problem is solved numerically by the method of successive approximations. It consists essentially of the determination of potential $\varphi=\varphi^{(k+1)}(x,y)$ in the $(k+1)$-st approximation by the second of Eqs. (3.2) on the basis of the distribution $q=q^{(k)}(x,y)$ in the k-th approximation. From Eqs. (3.1) with the use of $\varphi^{(k+1)}$ we obtain $u^{(k+1)}$ and $v^{(k+1)}$.

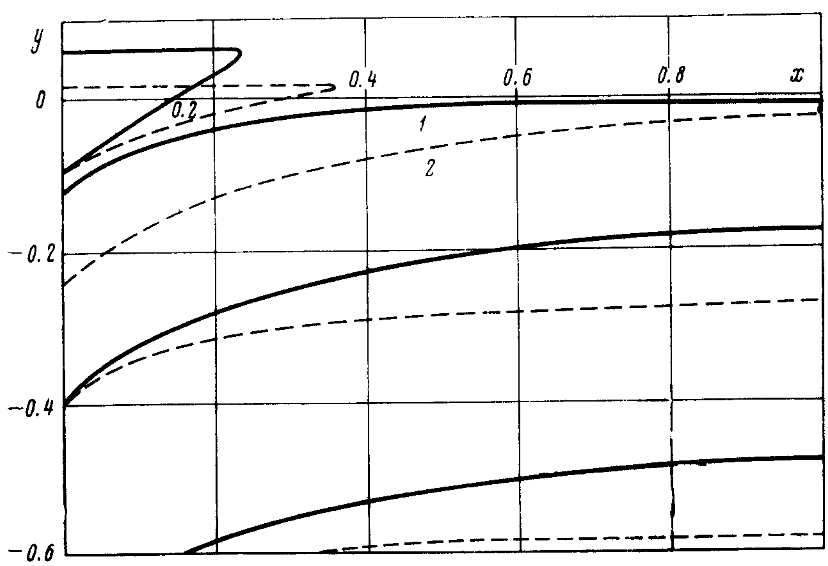


Fig. 2

Finally, from the first of Eqs. (3.2) we determine $q^{(k+1)}$. When $|q^{(k)}-q^{(k+1)}|<\varepsilon_q$ ($\varepsilon_q\ll 1$ is specified) the derivation of solution is considered completed.

The second of Eq. (3.2) is solved by the Seidel method of successive displacements with the speeding-up formula of L. A. Liusternik and subdivision of the integration region into rectangular cells. Equation (3.1) and the first of Eqs. (3.2) are integrated by the method of characteristics; the boundary of the zone containing a charge is determined by the method of straight characteristics, while u v and σ at inner nodes of the computation grid are obtained with the use of "inverse" characteristics.

The characteristics are represented by the charge streamlines

$$dy / dx = v / u \qquad (3.5)$$

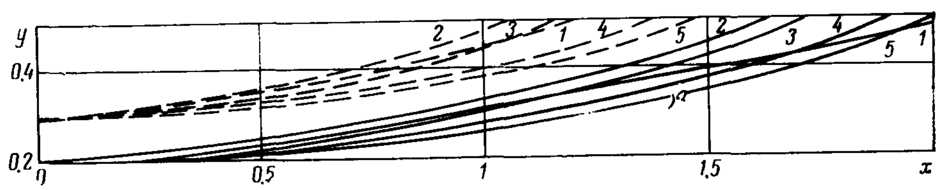
The related characteristic relationships for u, v and q are of the form

$$\frac{du}{dx} = \frac{1}{\tau u}\left(1 - u - \frac{\partial \varphi}{\partial x}\right), \quad \frac{dv}{dx} = \frac{1}{\tau u}\left(-v - \frac{\partial v}{\partial y}\right) \qquad (3.6)$$

$$\frac{dq}{dx} = -\frac{q}{u}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

Equations (3.5) and (3.6) are solved at each stage by the use of the iteration process. Individual ordinary differential equations are integrated by the Euler method followed by recalculation.

Below we present the results of computations for $h = 0.4$, $\tau = 1$, $\beta = 2$ and $0.4 \leqslant u_0 \leqslant 1.8$.



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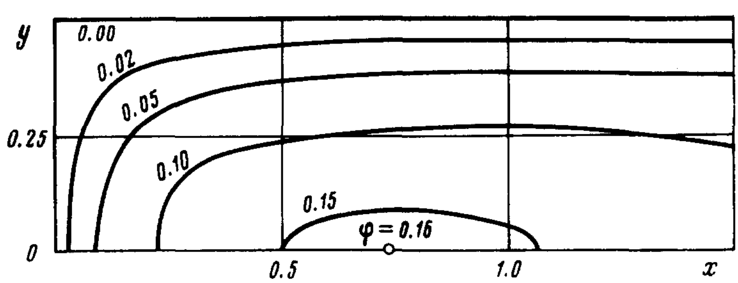

Fig. 4

The electric stream boundary $y = \Gamma(y)$ that separates region $y \leqslant \Gamma(x)$ with charged particles from region $y > \Gamma(x)$ where charges are absent, is shown in Fig. 3 for $u_0 = 0.4, 0.8, 1.2, 1.6$ and 1.8 by solid lines, and for $h = 0.6$ by dash lines.

Downstream the electric streams widen and eventually join the channel walls on which their particles are precipitated. The deflection of charged particles toward channel walls is caused by the action of induced transverse fields. If the moving particles are not charged, the flow is one-dimensional and the stream containing particles is of constant cross section. An increase of parameter β results in the increase of induced electric fields and a corresponding increased deflection of particles toward the stream periphery.

Equipotential lines ($u_0 = 1$, $h = 0.4$) which give an idea of the effect of electric forces on charged particles are shown in Fig. 4 in the plane of flow. The point corresponding to the highest φ lies on the stream axis at a distance, of the order of the channel width, from the initial cross section. As seen in Fig. 3, the initial increase of parameter u_0 results in a more intensive widening of the stream, because of the increase of the volume charge introduced into it and the consequent increase of the transverse electric field. There exists a certain u_0^* such that for $u_0 > u_0^*$ the electric stream begins to contract, which means that the initial longitudinal momentum begins to have a greater effect on the motion of charges.

The distribution of $q(x)$ along the stream

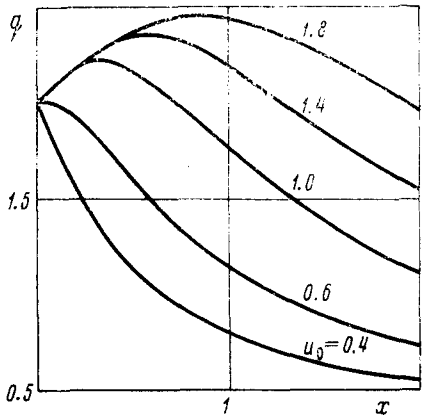

Fig. 5

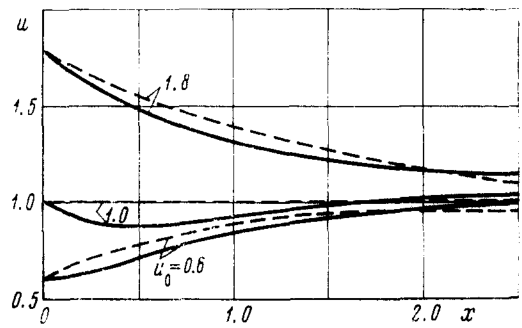

Fig. 6

axis is shown in Fig. 5 for several values of u_0. It is seen that function $q(x)$ is generally nonmonotonic. The region of decreasing (increasing) q along the initial section corresponds to acceleration (deceleration) of particles.

Distribution of particle longitudinal velocity along the channel axis for the same values of parameters are shown in Fig. 6 by solid lines. Similar curves for $\beta = 0$ (uncharged particles) are shown there by dash lines. The characteristic asymptotic tendency of u to the velocity of gas can be seen in the diagram. For $\beta \neq 0$ the change of particle velocity is nonmonotonic owing to the effect of the longitudinal electric field (see Fig. 4). Along the initial section it has a decelerating effect, while further downstream the velocity is accelerated. The over-all effect depends on the relation between friction and electric forces.

We note in concluding that the present investigation is a generalization of results obtained in [2 — 4] where the particular case of $\tau = 0$, which corresponds to the disregard of charged particle inertia, was considered. That condition is valid for light particles and ions.

The allowance for inertia results in a "lag" in the establishment of equilibrium values of the dynamic characteristics of gas. This explains the peculiar deformations of the electric stream shown above.

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ON THE PROPAGATION OF SMALL PERTURBATIONS IN A SONIC STREAM AND IN A QUIESCENT GAS

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Approximate equations are derived for small unsteady perturbations of a constant sonic stream and of quiescent gas. These equations, unlike the equation used for defining unstable transonic flows of gas, provide a correct definition of perturbation propagation from a point source in all directions [1].

1. Let us consider potential flows of perfect gas. Such flows are defined by the equation

$$(\Phi_t + V^2)_t + \Phi_x^2\Phi_{xx} + \Phi_y^2\Phi_{yy} + \Phi_z^2\Phi_{zz} + 2\Phi_x\Phi_y\Phi_{xy} + 2\Phi_x\Phi_z\Phi_{xz} + 2\Phi_y\Phi_z\Phi_{yz} = a^2(\Phi_{xx} + \Phi_{yy} + \Phi_{zz}) \quad (1.1)$$

$$a^2 = \rho^{\varkappa-1} = P^{(\varkappa-1)/\varkappa} = \frac{\varkappa+1}{2} - (\varkappa-1)\left(\Phi_t + \frac{1}{2}V^2\right)$$

$$V = (\Phi_x^2 + \Phi_y^2 + \Phi_z^2)^{1/2}$$

where Φ, x, y, z, t, V, a, P and ρ are, respectively, the dimensionless velocity potential, Cartesian coordinates, time, velocity of gas, speed of sound, pressure and density (related, respectively, to $a_*^2t_0$, a_*t_0, t_0, a_*, P_* and ρ_*, where the asterisk denotes parameters of the sonic stream $u = \Phi_x = a_*$ and $\Phi_y = 0$).

Let us consider transonic flows of gas, for which it is possible to use the linear theory

$$\Phi = x + \gamma\Phi_1 + \gamma^2\Phi_2 + \ldots, \quad \Phi_{1tt} + 2\Phi_{1xt} = \Phi_{1yy} + \Phi_{1zz} \quad (1.2)$$

$$\gamma \ll 1$$

However the linear theory has some shortcomings. The linear expansion (1.2) contains various irregularity regions for which the order of the second term $\gamma^2\Phi_2$ is the same as of the first $\gamma\Phi_1$.

As an example, we present two such expansions for one-dimensional flows

$$\Phi = x + \gamma\beta_1(\nu) + \gamma^2\left[-\frac{\varkappa+1}{8}\beta_1'^2(\nu)x + \beta_2(\nu)\right] + \cdots, \quad (1.3)$$

$$\nu = 2t - x$$

$$\Phi = x + \gamma\alpha_1(x) + \gamma^2\left[-\frac{\varkappa+1}{8}\alpha_1'^2(x)\nu + \alpha_2(x)\right] + \cdots \quad (1.4)$$